

Effective electric and magnetic polarizabilities of pointlike spin-1/2 particles

A. J. Silenko*

*Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, Dubna 141980, Russia
Research Institute for Nuclear Problems,
Belarusian State University, Minsk 220030, Belarus*

Abstract

Effective electric and magnetic polarizabilities of pointlike spin-1/2 particles possessing an anomalous magnetic moment are calculated with the transformation of an initial Hamiltonian to the Foldy-Wouthuysen representation. Polarizabilities of spin-1/2 and spin-1 particles are compared.

PACS numbers: 03.65.Pm, 11.10.Ef, 12.20.Ds

*Electronic address: alsilenko@mail.ru

In this work, the Foldy-Wouthuysen (FW) transformation is used to determine the electric and magnetic polarizabilities of pointlike spin-1/2 particles possessing an anomalous magnetic moment (AMM).

The unique properties of the FW representation [1] make it a very convenient tool for transition to semiclassical approximation and finding the classical limit of relativistic quantum mechanics. Even for relativistic particles in an external field, the operators in this representation are completely analogous to the corresponding operators of nonrelativistic quantum mechanics. In particular, the localization operators (the Newton-Wigner operators) and the momentum operators are equal to [2] \mathbf{r} and $\mathbf{p} = -i\hbar\nabla$, and the polarization operator for the spin-1/2 particles is the Dirac matrix $\mathbf{\Pi}$. In other representations, these operators are defined by considerably more cumbersome formulas (see [1, 3]). Apart from the simple and unambiguous form of operators that correspond to the classical observed ones, the most significant merit of FW representation is the restoration of the probabilistic interpretation of the wave function. Since, as was said above, it is in the FW representation that the Newton-Wigner operator, which characterizes the location of a particles geometric center, is equal to the radius-vector \mathbf{r} , the squared wave function modulus determines the probability density of the particles location at the point with the given radius vector. We should note that, in the FW representation, the Hamiltonian and all operators are diagonal over two spinors (block-diagonal). Using this representation eliminates the possibility of ambiguous solutions for the problem of finding the classical limit of relativistic quantum mechanics [1, 4].

The most essential parameters that characterize the particles and the nuclei are scalar electric and magnetic polarizabilities. Their contribution to the Hamiltonian in the FW representation is determined by the expression

$$\Delta\mathcal{H}_{FW} = -\frac{1}{2}\alpha_s E^2 - \frac{1}{2}\beta_s B^2. \quad (1)$$

These parameters are *effective* polarizabilities which origin is purely quantum mechanical. In particular, any real separation of electric charges of pointlike particles do not take place.

We consider here a case of stationary and uniform electric (\mathbf{E}) and magnetic (\mathbf{B}) fields and use the system of units $\hbar = 1$, $c = 1$.

While in the classical physics a particle may have nonzero polarizabilities only provided that it has an internal structure, in quantum mechanics even pointlike objects appear to

have nonzero polarizabilities. These parameters can be defined by the FW transformation of the initial DiracPauli equation [5] for particles with an AMM followed by transition to the classical limit. Solving this problem necessitates, however, the calculation of external-field quadratic summands. This procedure requires caution, since different techniques yield different results (see [6] and references therein). Correct results are obtained by the Eriksen method [7]. It is convenient to divide the initial DiracPauli Hamiltonian [5] into the even and odd summands that commute and anticommute with the Dirac matrix β , respectively:

$$\mathcal{H}_D = \beta m + \mathcal{E} + \mathcal{O}, \quad \beta \mathcal{E} = \mathcal{E} \beta, \quad \beta \mathcal{O} = -\mathcal{O} \beta. \quad (2)$$

Here,

$$\mathcal{E} = e\Phi - \mu' \mathbf{\Pi} \cdot \mathbf{B}, \quad \mathcal{O} = c\boldsymbol{\alpha} \cdot \boldsymbol{\pi} + i\mu' \boldsymbol{\gamma} \cdot \mathbf{E}, \quad (3)$$

where μ' is the AMM. We use the conventional denotations [8] for the Dirac matrices.

The expansion in $1/m$ powers of the Hamiltonian in the FW representation obtained by the Eriksen method is presented in Refs. [6, 9]. In the case under consideration, the summands proportional to the fourth and higher powers of the reciprocal mass can be neglected. In this case, the Hamiltonian in the FW representation is defined according to the equation

$$\mathcal{H}_{FW} = \beta \left(m + \frac{\mathcal{O}^2}{2m} - \frac{\mathcal{O}^4}{8m^3} \right) + \mathcal{E} - \frac{1}{8m^2} [\mathcal{O}, [\mathcal{O}, \mathcal{F}]] + \frac{\beta}{16m^3} \{ \mathcal{O}, [[\mathcal{O}, \mathcal{F}], \mathcal{F}] \}, \quad (4)$$

where $\mathcal{F} = \mathcal{E} - i\partial/\partial t$. For the considered stationary problem, $\mathcal{F} = \mathcal{E}$.

Calculation according to Eqs. (3) and (4) yields the following expression:

$$\begin{aligned} \mathcal{H}_{FW} = & \beta \left(m + \frac{\pi^2}{2m} - \frac{\pi^4}{8m^3} \right) + e\Phi + \frac{1}{2m} \left(\frac{\mu_0}{2} + \mu' \right) (2\boldsymbol{\Sigma} \cdot [\boldsymbol{\pi} \times \mathbf{E}] - \nabla \cdot \mathbf{E}) \\ & - (\mu_0 + \mu') \mathbf{\Pi} \cdot \mathbf{B} + \frac{\mu'}{4m^2} \{ \mathbf{\Pi} \cdot \boldsymbol{\pi}, \boldsymbol{\pi} \cdot \mathbf{B} \} + \beta \frac{(\mu_0 + \mu')\mu'}{2m} E^2 - \beta \frac{\mu_0^2}{2m} B^2, \end{aligned} \quad (5)$$

where $\mu_0 = e/(2m)$ is the Dirac magnetic moment.

We should note that the last summand in Eq. (4) does not make any contribution to the electric and magnetic polarizabilities. Calculations performed by the method developed by Foldy and Wouthuysen [1] and by other iterative methods (see Refs. [9, 10] and references therein) lead to a different form of this summand and, as a consequence, do not yield the correct expression for the electric polarizability. A comparison of Eqs. (1) and (5) shows that the scalar electric and magnetic polarizabilities of the pointlike particles possessing an

AMM have the form

$$\alpha_S = -\frac{(\mu_0 + \mu')\mu'}{m} = -\frac{e^2 g(g-2)}{16m^3}, \quad \beta_S = \frac{\mu_0^2}{m} = \frac{e^2}{4m^3}. \quad (6)$$

The matrix β can be omitted since, in the FW representation, the lower spinor is equal to zero.

A comparison of polarizabilities of pointlike spin-1/2 and spin-1 particles is important. The spin-1 particles are characterized by not only scalar but also tensor polarizabilities which are calculated in Ref. [11]. The scalar polarizabilities of such particles are defined by the expressions [11]:

$$\alpha_S = -\frac{e^2(g-1)^2}{m^3}, \quad \beta_S = 0. \quad (7)$$

Thus, the scalar magnetic polarizability of the spin-1 particles equals zero and the scalar electric polarizability is nonzero for particles with not only an anomalous but also normal ($g = 2$) magnetic moment. These properties differ from the corresponding properties of the spin-1/2 particles.

We should note that the scalar polarizabilities of pointlike spin-0 particles are equal to zero (see Ref. [12]).

The scalar polarizabilities of pointlike particles belong to quantum mechanical effects. Such particles do not have any charge distribution and their interaction with the electric field does not lead to any charge separation.

The work was supported by the Belarusian Republican Foundation for Fundamental Research (Grant No. $\Phi 14D-007$).

-
- [1] L. L. Foldy, S. A. Wouthuysen, "On the Dirac Theory of Spin 1/2 Particles and Its Non-Relativistic Limit," Phys. Rev. **78**, 29-36 (1950).
 - [2] T. D. Newton, E. P. Wigner, "Localized States for Elementary Systems," Rev. Mod. Phys. **21**, 400-406 (1949).
 - [3] A. J. Silenko, "Foldy-Wouthuysen transformation for relativistic particles in external fields," J. Math. Phys. **44**, 2952-2966 (2003).
 - [4] J. P. Costella, B. H. J. McKellar, "The Foldy-Wouthuysen transformation," Am. J. Phys. **63**, 1119-1121 (1995).

- [5] W. Pauli, “Relativistic Field Theories of Elementary Particles,” *Rev. Mod. Phys.* **13**, 203-232 (1941).
- [6] E. de Vries, “Foldy-Wouthuysen Transformations and Related Problems” *Fortschr. Phys.* **18**, 149-182 (1970).
- [7] E. Eriksen, “Foldy-Wouthuysen Transformation. Exact Solution with Generalization to the Two-Particle Problem,” *Phys. Rev.* **111**, 1011-1016 (1958).
- [8] V. B. Berestetskii, E. M. Lifshitz, L. P. Pitaevskii, *Quantum Electrodynamics*, 2nd ed. (Butterworth-Heinemann, Oxford, 1982).
- [9] E. de Vries, J. E. Jonker, “Non-relativistic approximations of the Dirac Hamiltonian,” *Nucl. Phys. B* **6**, 213-225 (1968); A. J. Silenko, “Comparative analysis of direct and step-by-step Foldy-Wouthuysen transformation methods,” *Theor. Math. Phys.* **176**, 987999 (2013).
- [10] V. P. Neznamov, A. J. Silenko, “Foldy-Wouthuysen wave functions and conditions of transformation between Dirac and Foldy-Wouthuysen representations,” *J. Math. Phys.* **50**, 122302 (2009).
- [11] A. J. Silenko, “Quantum-mechanical description of spin-1 particles with electric dipole moments,” *Phys. Rev. D* **87**, 073015 (2013).
- [12] A. J. Silenko, “The Hamiltonian operator and the quasi-classical limit for scalar particles in an electromagnetic field,” *Theor. Math. Phys.* **156**, 390-411 (2008).